405/Math

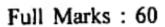
UG/3rd Sem/MATH(H)CC-05-T/19

U.G. 3rd Semester Examination - 2019

MATHEMATICS

[HONOURS]

Course Code: MATH(H)CC-05-T



Time: $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols and notations have their usual meanings.

Answer any ten questions:

 $2 \times 10 = 20$

Use sequential criterion for limits to show that the following limit does not exist

$$\lim_{x\to 0}\frac{1}{x}\sin\frac{1}{x}.$$

- Give example of function f and g which are not continous at a point c∈ R but the sum ftg is continous as c.
 - ii) Using ε-δ definition, show that

$$\lim_{x \to \infty} \frac{[x]}{x} = 1.$$

[Turn over]

- Xiv) Verify what her (IR, d) is a metric space, where $d(x, y) = |x^i - y^i|, \forall x, y \in \mathbb{R}$
- Define diameter of a set in a metric space (x, d).
- Does f'(c) = 0 always imply existence of an extremum of f at c? Justify.
 - vi Give an example of a function which has a jump discontinuity in its domain of definition.
 - wiii) Show that the equation $f(x)=xe^{x}-2$ has a root in [0, 1].
 - ix) Expand log sin (x+h) in power of h by Taylor's Theorem.
 - Give geometrical interpretation of Lagrange's 入x) Mean Value Theorem.
 - Define limit point of a set in a metric space (x, d). Give one example.
 - xii) For a metric space X, show that a point a ∈ X is a cluster point of $A \subset X$ if there exists $\{a_*\}_{*-1}^n$ in A such that Lt $a_n = a$.

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- xifi) Show that there does not exist a function & such that $\phi'(x) = f(x)$, where f(x) = x - [x], $x \in [0, 2]$.
- xiv) Discuss the applicability of Rolle's theorem for $f(x) = 2 + (x-1)^{\frac{1}{2}}$ in [0, 2].
- xv) Show that f(x)=|x+2| is continuous at x=-2 but not differentiable at this point.
- Answer any four questions:

5-4-20

Let f be a continuous function on [a, b] and c be any real number between f(a) and f(b), then show that there exist a real number x in (a, b) such that f(x)=c.

> Construct an example to show that continuity of f is not necessary for the existence of such x 3+2 as above.

- State and prove Rolle's theorem.
- iii) Find the maxima and minima of the function $f(x) = \sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x$ for all $x \in [0, \pi]$.

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[Turn over]

iv) Define a Metric space show that (R², d) is a metric space, where the metric d:R²×R²→R is defined by d(x, y) = |x₁ - y₁| + |x₂ - y₂|;
 x,y∈R² when x=(x₁, x₂), y=(y₁, y₂).

Suppose n-th derivative of a function f exists finitely in a closed interval [a, a+h]. Then show that there exists a positive proper fraction θ satisfying the relation

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + ...$$
$$+ \frac{h^{n-1}}{(n-1)!}f^{n-1}(a) + \frac{h^n}{n!}f^n(a+\theta h).$$

vi) Let $f: [a,b] \to \mathbb{R}$ be such that f has a local extremum as an interior point c of [a,b]. If f'(c) exists, then prove that f'(c)=0.

Answer any two questions from Question No. 3 to Question No. 6: 10×2=20

- A function f: [0, 1] → [0, 1] is continous on [0,1].
 Prove that there exists a point c in [0,1] such that f(c)=c.
 - b) Show that every uniformly continous function on an interval is continous on that interval, but the converse is not true.
 - Prove that the function $f(x) = \frac{1}{x}$, $x \in (0, 1]$ is not uniformly continous on (0, 1].
- f. a) If f' and g' exist for all $x \in [a, b]$ and $g'(x) \neq 0 \ \forall \ x \in (a, b)$, then prove that for some $c \in (a, b)$, $\frac{f(c) f(a)}{g(b) g(c)} = \frac{f'(c)}{g'(c)}$.
 - by Obtain the Maclaurin's series expansion of log(1+x), -1 < x ≤ 1.</p>

(5)

Show that

$$x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}, x > 0.$$

3+4+3

- a) Prove that in a metric space every open ball is an open set and every closed ball is a closed set.
 - b) Define the following with example:
 - i) Subspace of a metric space
 - ii) Separable metric space (3+3)+(2+2)
- 6. a) Show that the function f on [0, 1] defined as $f(x) = \frac{1}{2^n} \text{ when } \frac{1}{2^{n+1}} < x \le \frac{1}{2^n}, \ n = 0, 1, 2, ...,$ $f(0) = 0 \text{ is discontinuous at } \frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3,$
 - b) Show that $\lim_{n\to\infty} a^n \cdot \sin \frac{b}{a^n} = \begin{cases} 0 & \text{if } 0 < a < 1 \\ b & \text{if } a > 1 \end{cases}$.

c) Prove that in a metric space (X, d), the interior of a set A ⊂ X is the largest open subset of A.
3+3+4

